SL Paper 1

Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

a. Find $\cos\theta$. [3]

b. Find $\tan 2\theta$. [5]

Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

In the following table, $f'\left(\frac{\pi}{2}\right) = p$ and $f''\left(\frac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \frac{\pi}{2}$.

x	$\frac{\pi}{2} - 0.1$	$\frac{\pi}{2}$	$\frac{\pi}{2} + 0.1$
$f'(x)$	-1.01	p	-1.01
$f''(x)$	0.203	q	-0.203

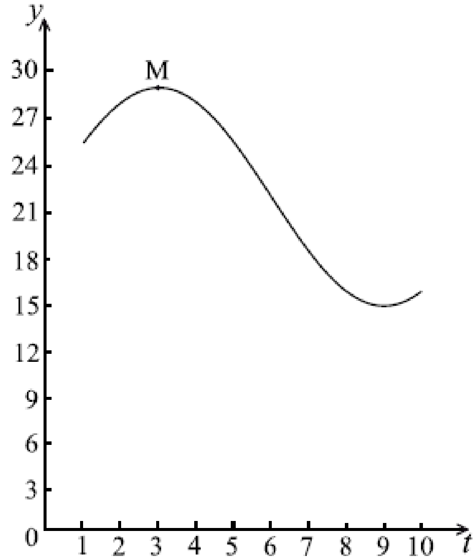
a. Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$. [5]

b. Find $f''(x)$. [3]

c. Find the value of p and of q. [3]

d. Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \frac{\pi}{2}$. [2]

Let $f(t) = a\cos b(t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.

When $t = 9$, there is a minimum value of 15.

a(i), (ii), (iii) and (iv). [7]

(i) Find the value of a.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

b. The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$. [2]

Let M' be the image of M under P. Find the coordinates of M' .

c. The graph of g is the image of the graph of f under P. [4]

Find $g(t)$ in the form $g(t) = 7\cos B(t - c) + D$.

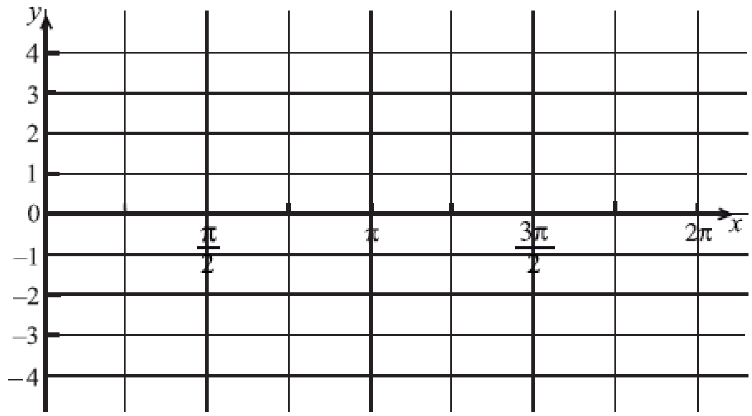
d. The graph of g is the image of the graph of f under P. [3]

Give a full geometric description of the transformation that maps the graph of g to the graph of f .

Consider $g(x) = 3\sin 2x$.

a. Write down the period of g. [1]

b. On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$. [3]

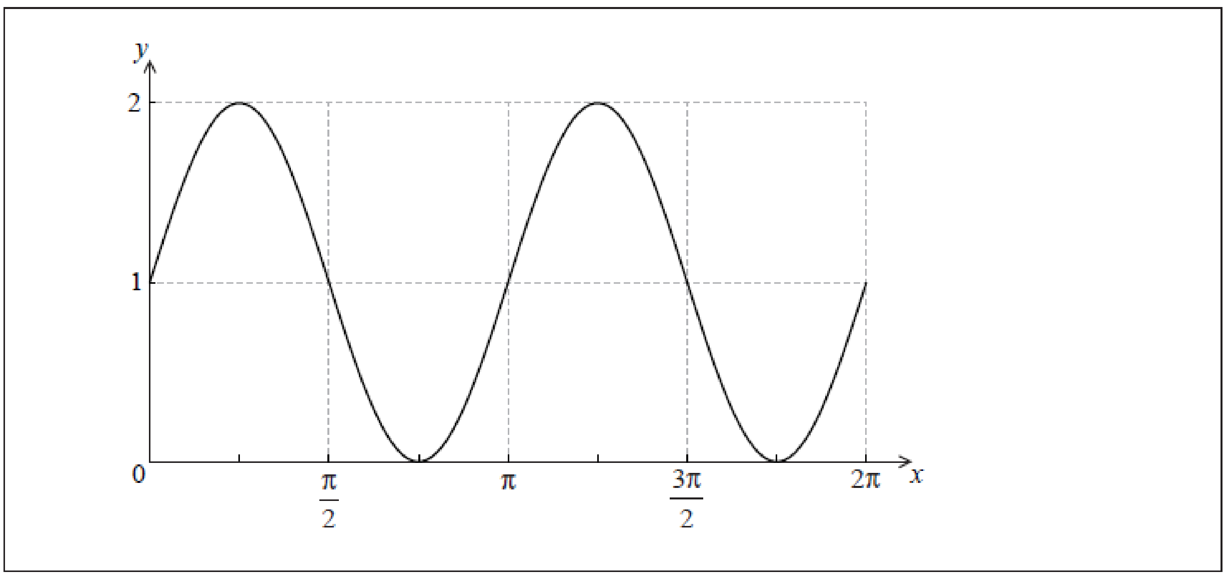


c. Write down the number of solutions to the equation $g(x) = 2$, for $0 \le x \le 2\pi$. [2]

Let $f(x) = (\sin x + \cos x)^2$.

a. Show that $f(x)$ can be expressed as $1 + \sin 2x$. [2]

b. The graph of *f* is shown below for $0 \le x \le 2\pi$. [2]



Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of *g* for $0 \le x \le 2\pi$.

c. The graph of *g* can be obtained from the graph of *f* under a horizontal stretch of scale factor *p* followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$. [2]

Write down the value of *p* and a possible value of *k* .

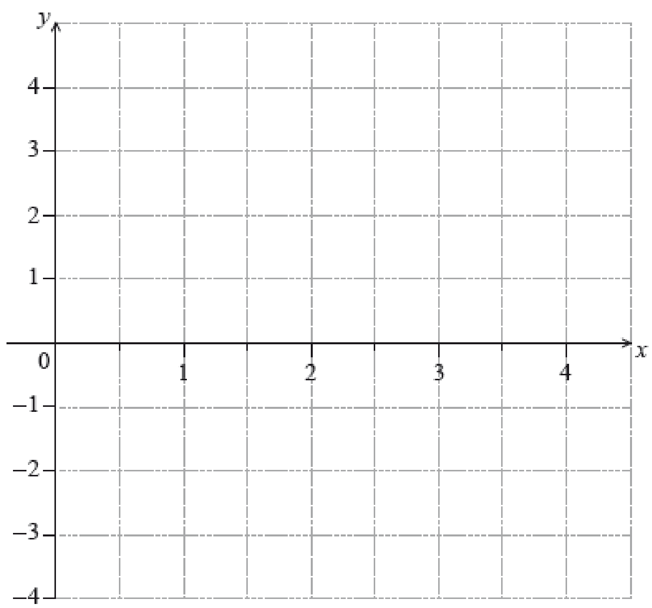
Let $f(x) = \sqrt{3}e^{2x} \sin x + e^{2x} \cos x$, for $0 \le x \le \pi$. Solve the equation $f(x) = 0$.

Let $f(x) = 3\sin\left(\frac{\pi}{2}x\right)$, for $0 \leqslant x \leqslant 4$.

a. (i) Write down the amplitude of f. [3]

(ii) Find the period of f.

b. On the following grid sketch the graph of f. [4]



Given that $\sin x = \frac{3}{4}$, where x is an obtuse angle,

a. find the value of $\cos x$; [4]

b. find the value of $\cos 2x$. [3]

Note: In this question, distance is in metres and time is in seconds.

Two particles P_1 and P_2 start moving from a point A at the same time, along different straight lines.

After t seconds, the position of P_1 is given by $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

Two seconds after leaving A, P_1 is at point B.

Two seconds after leaving A, P_2 is at point C, where $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

a. Find the coordinates of A. [2]

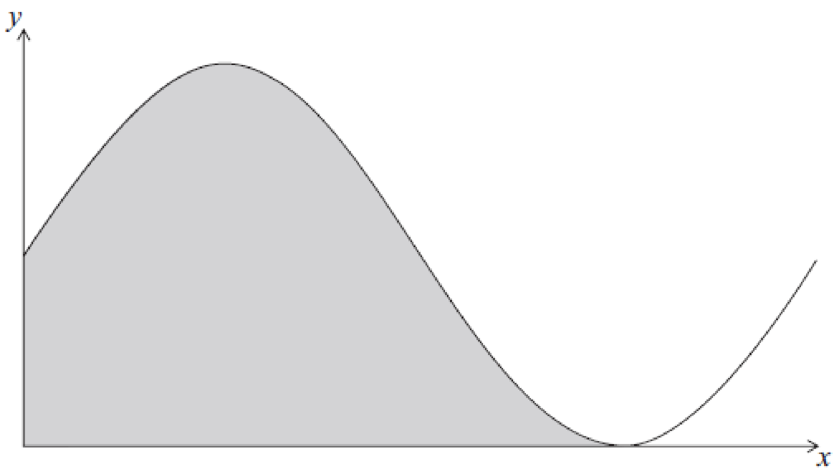
b.i. Find $\overrightarrow{\mathrm{AB}}$; [3]

b.ii. Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

c. Find $\cos \mathrm{B\hat{A}C}$. [5]

d. Hence or otherwise, find the distance between P_1 and P_2 two seconds after they leave A. [4]

Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of f , the x-axis, and the y-axis.

a(i) and (ii) Solve for $0 \le x < 2\pi$ [5]

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$.

b. Write down the exact value of the x-intercept of f , for $0 \le x < 2\pi$. [1]

c. The area of the shaded region is k . Find the value of k , giving your answer in terms of π . [6]

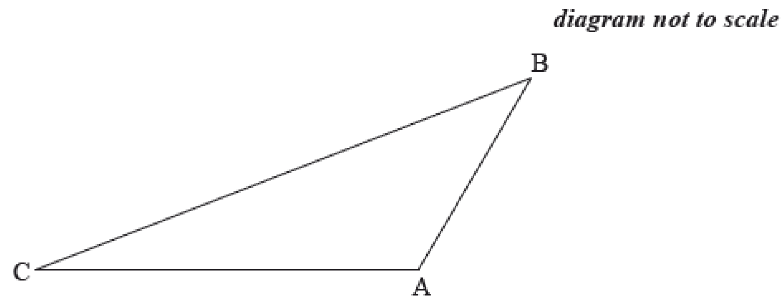
d. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [2]

Give a full geometric description of this transformation.

e. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g. [3]

Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of p.

The following diagram shows triangle ABC.



Let $\overrightarrow{AB} \bullet \overrightarrow{AC} = -5\sqrt{3}$ and $\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right| = 10$. Find the area of triangle ABC.

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \le x \le 2\pi$.

Let $f : x \mapsto \sin^3 x$.

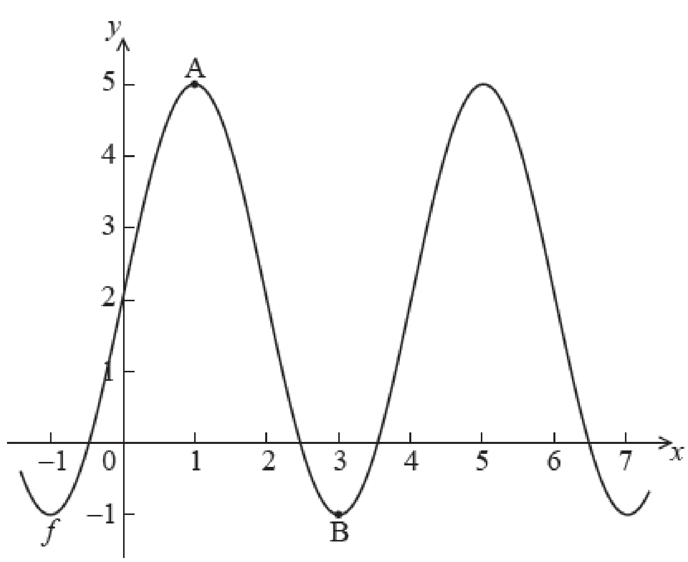
a. (i) Write down the range of the function f. [5]

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

b. Find $f'(x)$, giving your answer in the form $a\sin^p x\cos^q x$ where $a,\ p,\ q \in \mathbb{Z}$. [2]

c. Let $g(x) = \sqrt{3}\sin x(\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis. [7]

The diagram below shows part of the graph of a function f .



The graph has a maximum at A(1, 5) and a minimum at B(3, −1) .

The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

. (a) p [6]

(b) q

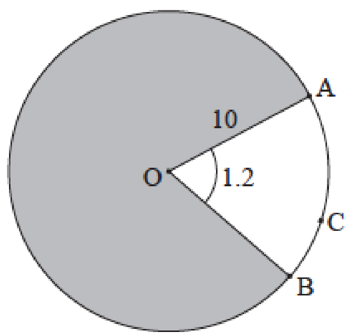
(c) r .

a. p [2]

b. q [2]

c. r . [2]

The following diagram shows a circle with centre O and a radius of 10 cm. Points A, B and C lie on the circle.



Angle AOB is 1.2 radians.

a. Find the length of arc ACB. [2]

b. Find the perimeter of the shaded region. [3]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$P(A = a)$	$\cos\theta$	$2\cos 2\theta$

a. Show that $\cos\theta = \frac{3}{4}$. [6]

b. Given that $\tan\theta > 0$, find $\tan\theta$. [3]

c. Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of $y$between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

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a. Show that $4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$. [2]

b. **Hence**, solve the equation $4 - \cos 2\theta + 5\sin\theta = 0$ for $0 \le \theta \le 2\pi$. [5]

Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the x-axis.

a. Write down the value of $\tan\theta$. [1]

b(i) and (ii). Find the value of [6]

(i) $\sin 2\theta$;

(ii) $\cos 2\theta$.

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

a. Find $\cos\theta$. [3]

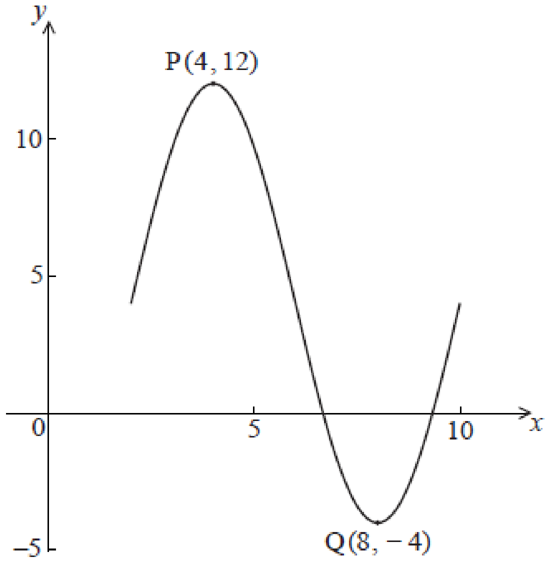
b. Find $\cos 2\theta$. [2]

Let $f(x) = \sin^3 x + \cos^3 x \tan x,\ \frac{\pi}{2} < x < \pi$.

a. Show that $f(x) = \sin x$. [2]

b. Let $\sin x = \frac{2}{3}$. Show that $f(2x) = -\frac{4\sqrt{5}}{9}$. [5]

The following diagram shows the graph of $f(x) = a\sin(b(x - c)) + d$, for $2 \le x \le 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4) .

a(i), (ii) and (iii). Use the graph to write down the value of [3]

(i) a ;

(ii) c ;

(iii) d .

b. Show that $b = \frac{\pi}{4}$. [2]

c. Find $f'(x)$. [3]

d. At a point R, the gradient is -2π . Find the x-coordinate of R. [6]

The expression $6 \sin x \cos x$ can be expressed in the form $a \sin bx$.

a. Find the value of a and of b . [3]

b. Hence or otherwise, solve the equation $6 \sin x \cos x = \frac{3}{2}$, for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$. [4]

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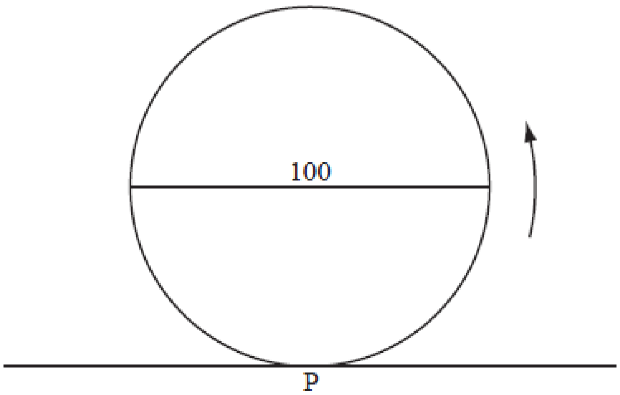
a. Let $\sin 100^\circ = m$. Find an expression for $\cos 100^\circ$ in terms of m. [3]

b. Let $\sin 100^\circ = m$. Find an expression for $\tan 100^\circ$ in terms of m. [1]

c. Let $\sin 100^\circ = m$. Find an expression for $\sin 200^\circ$ in terms of m. [2]

Let $\int_{\pi}^{a} \cos 2x \mathrm{d}x = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a.

The following diagram represents a large Ferris wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counter-clockwise) direction. One revolution takes 20 minutes.

Let $h(t)$ metres be the height of P above ground level after t minutes. Some values of $h(t)$ are given in the table below.

t	$h(t)$
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

a(i) and (ii). Write down the height of P above ground level after [2]

(i) 10 minutes;

(ii) 15 minutes.

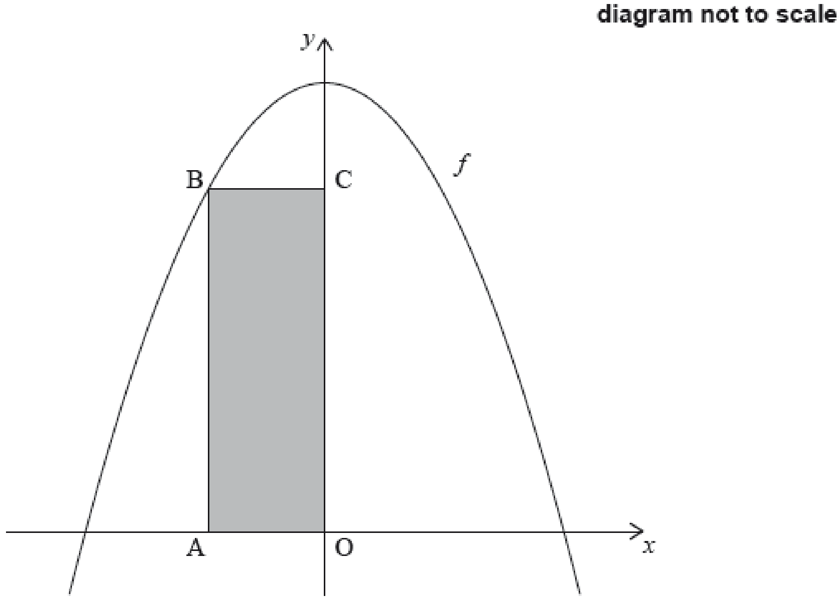
b(i) and (ii). (i) Show that $h(8) = 90.5$. [4]

(ii) Find $h(21)$.

c. **Sketch** the graph of h , for $0 \leq t \leq 40$. [3]

d. Given that h can be expressed in the form $h(t) = a \cos bt + c$, find a , b and c . [5]

Let $f(x) = 15 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis.



Find the x-coordinate of A that gives the maximum area of OABC.

Let $f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of f passes through the point $\left(\frac{\pi}{4}, 6\right)$.

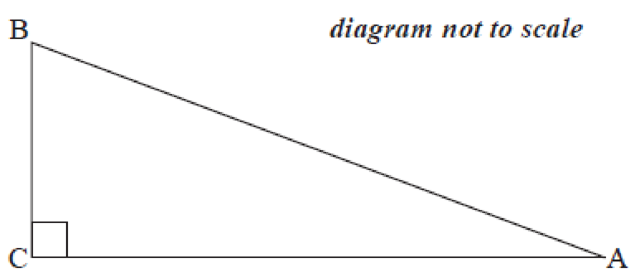
a. Find the value of k. [3]

b. Find the minimum value of $f(x)$. [2]

c. Let $g(x) = \sin x$. The graph of g is translated to the graph of f by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$. [2]

Write down the value of p and of q.

The following diagram shows a right-angled triangle, ABC, where $\sin \mathrm{A} = \frac{5}{13}$.



a. Show that $\cos A = \frac{12}{13}$. [2]

b. Find $\cos 2A$. [3]

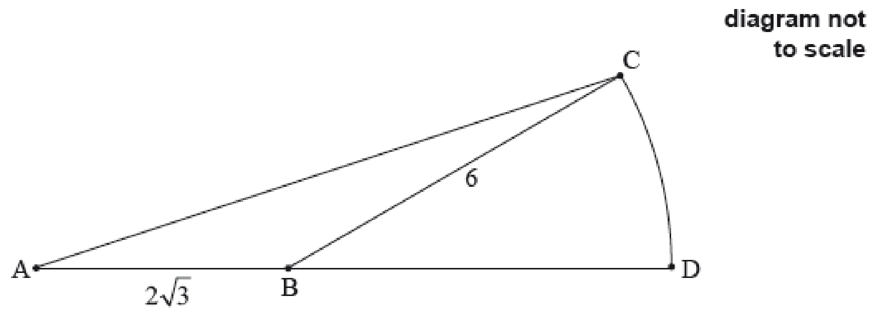
Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

a. Find $f\left(\frac{\pi}{2}\right)$. [2]

b. Find $(g \circ f)\left(\frac{\pi}{2}\right)$. [2]

c. Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. [3]

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius 6 cm. The points A , B and D are on the same line.



$\text{AB} = 2\sqrt{3}\text{ cm}, \text{BC} = 6\text{ cm}, \text{area of triangle ABC} = 3\sqrt{3}\text{ cm}^2, \text{A}\hat{\text{B}}\text{C}$ is obtuse.

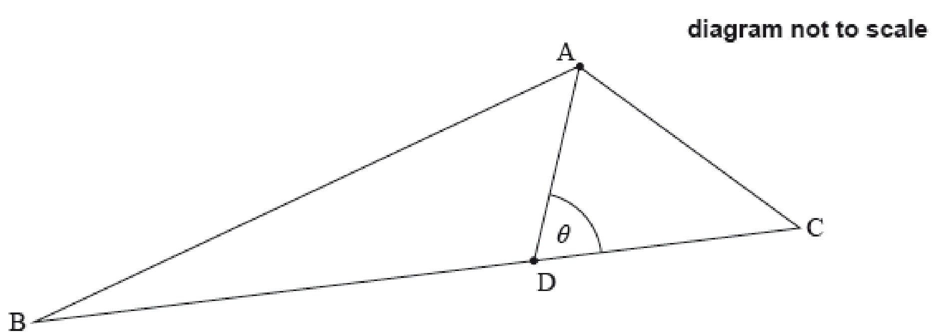
a. Find $\text{A}\hat{\text{B}}\text{C}$. [5]

b. Find the exact area of the sector BDC. [3]

Let $\overrightarrow{\text{OA}} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\overrightarrow{\text{OB}} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

The point C is such that $\overrightarrow{\text{AC}} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $\text{ADC} = \theta$.



a. (i) Find $\overrightarrow{\mathrm{AB}}$. [4]

(ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$.

b. Show that the coordinates of C are $(-2,\ 1,\ 3)$. [1]

c. Write down an expression in terms of θ for [2]

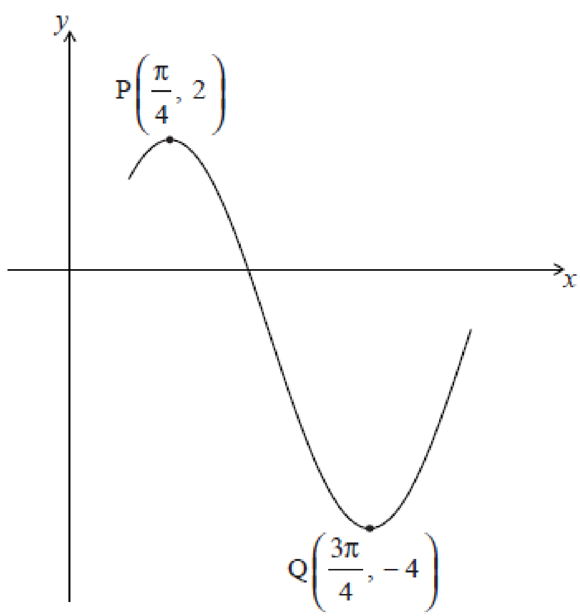
(i) angle ADB;

(ii) area of triangle ABD.

d. Given that $\frac{\text{area } \Delta\text{ABD}}{\text{area } \Delta\text{ACD}} = 3$, show that $\frac{\text{BD}}{\text{BC}} = \frac{3}{4}$. [5]

e. Hence or otherwise, find the coordinates of point D. [4]

The diagram below shows part of the graph of $f(x) = a\cos(b(x-c)) - 1$, where $a > 0$.



The point P $\left(\frac{\pi}{4}, 2\right)$ is a maximum point and the point Q $\left(\frac{3\pi}{4}, -4\right)$ is a minimum point.

a. Find the value of a . [2]

b(i) and (ii). (i) Show that the period of f is π . [4]

(ii) Hence, find the value of b .

c. Given that $0 < c < \pi$, write down the value of c . [1]

Let $p = \sin 40^\circ$ and $q = \cos 110^\circ$. Give your answers to the following in terms of p and/or q .

a(i) and (ii). Write down an expression for [2]

(i) $\sin 140^\circ$;

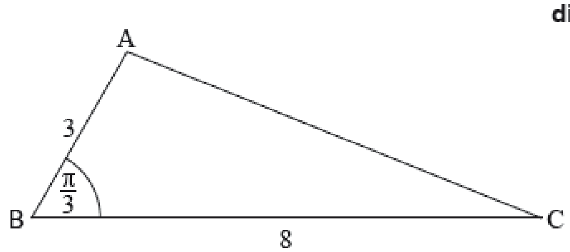
(ii) $\cos 70^\circ$.

b. Find an expression for $\cos 140^\circ$. [3]

c. Find an expression for $\tan 140^\circ$. [1]

The following diagram shows triangle ABC, with $\mathrm{AB} = 3$ cm, $\mathrm{BC} = 8$ cm, and $\mathrm{A\hat{B}C} = \frac{\pi}{3}$.

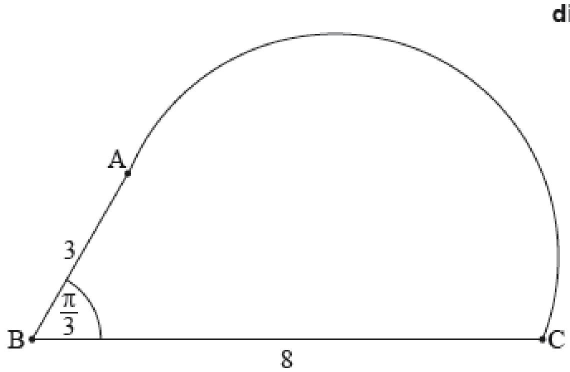
diagram not to scale



a. Show that $\mathrm{AC} = 7$ cm. [4]

b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle. [3]

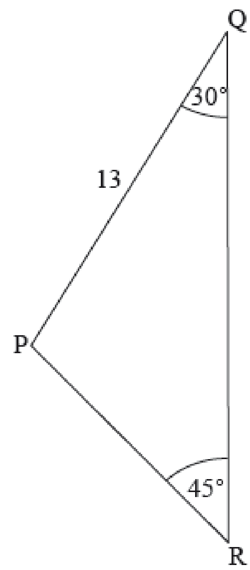
diagram not to scale



Find the exact perimeter of this shape.

The following diagram shows triangle PQR.

diagram not to scale



$\hat{PQR} = 30°$, $\hat{QRP} = 45°$ and $PQ = 13$ cm.

Find PR.

Let $f(x) = e^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

a. Write down [2]

(i) $f'(x)$;

(ii) $g'(x)$.

b. Let $h(x) = e^{-3x} \sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. [4]

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a. Given that $\cos A = \frac{1}{3}$ and $0 \le A \le \frac{\pi}{2}$, find $\cos 2A$. [3]

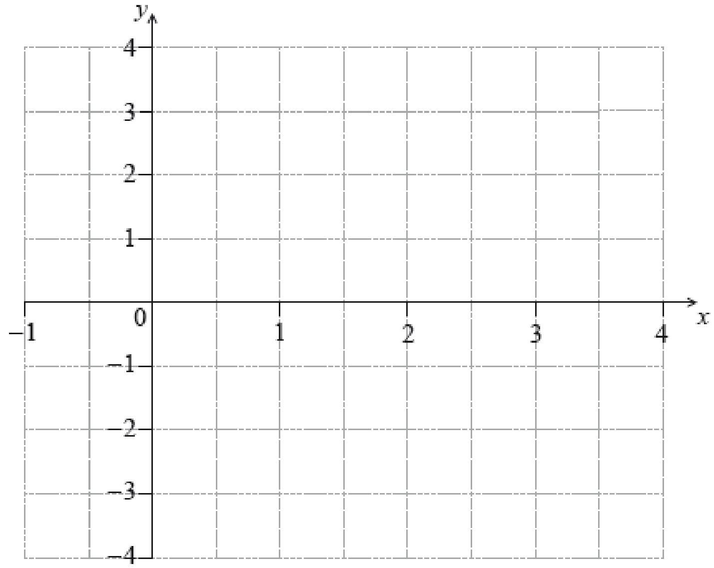
b. Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\cos B$. [3]

Let $f(x) = 3\sin(\pi x)$.

a. Write down the amplitude of f. [1]

b. Find the period of f. [2]

c. On the following grid, sketch the graph of $y = f(x)$, for $0 \le x \le 3$. [4]



The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

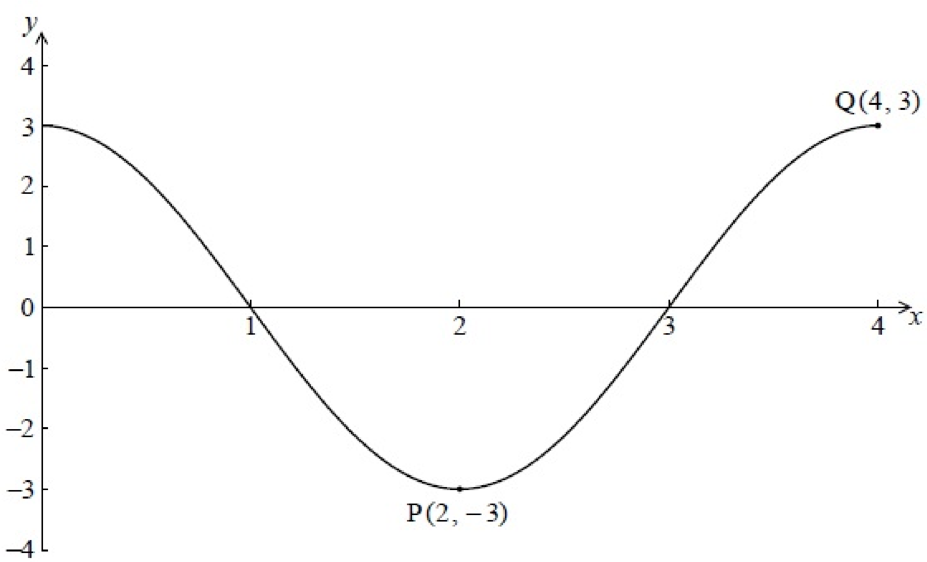
a.i. Find an expression for *r* in terms of *θ*. [2]

a.ii. Find the possible values of *r*. [3]

b. Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. [4]

c. Find the values of *θ* which give the greatest value of the sum. [6]

The following diagram shows the graph of $f(x) = a\cos(bx)$, for $0 \le x \le 4$.



There is a minimum point at P(2, − 3) and a maximum point at Q(4, 3) .

a(i) and (ii) (i) Write down the value of a . [3]

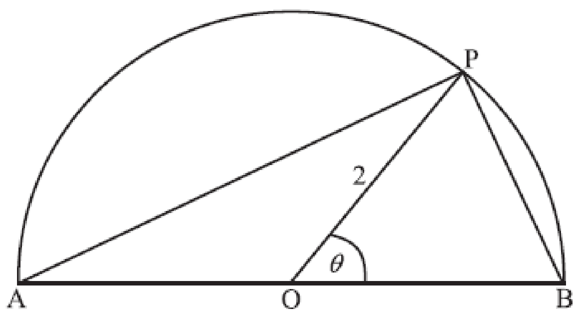
(ii) Find the value of b .

b. Write down the gradient of the curve at P. [1]

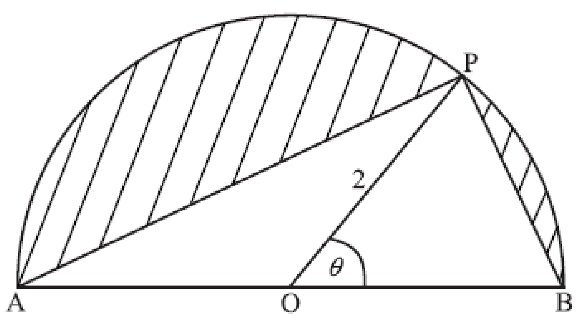
c. Write down the equation of the normal to the curve at P. [2]

The following diagram shows a semicircle centre O, diameter [AB], with radius 2.

Let P be a point on the circumference, with $\mathrm{P\hat{O}B} = \theta$ radians.



Let S be the total area of the two segments shaded in the diagram below.



a. Find the area of the triangle OPB, in terms of θ . [2]

b. Explain why the area of triangle OPA is the same as the area triangle OPB. [3]

c. Show that $S = 2(\pi - 2\sin\theta)$. [3]

d. Find the value of θ when S is a local minimum, justifying that it is a minimum. [8]

e. Find a value of θ for which S has its greatest value. [2]

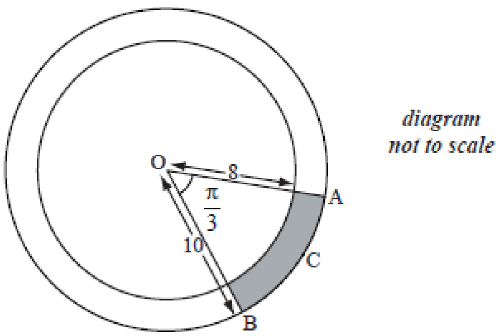
Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leqslant x \leqslant 1$, and $g(x) = \cos(x)$, for $0 \leqslant x \leqslant \pi$.

Let $h(x) = (f \circ g)(x)$.

a. Write $h(x)$ in the form $a\sin(bx)$, where $a,\ b \in \mathbb{Z}$. [5]

b. Hence find the range of h. [2]

The diagram shows two concentric circles with centre O.



The radius of the smaller circle is 8 cm and the radius of the larger circle is 10 cm.

Points A, B and C are on the circumference of the larger circle such that $\text{A}\hat{\text{O}}\text{B}$ is $\frac{\pi}{3}$ radians.

a. Find the length of the arc ACB . [2]

b. Find the area of the shaded region. [4]

The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{\text{OP}} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix},\ \overrightarrow{\text{OQ}} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{\text{OR}} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$

a. Find [3]

(i) $\overrightarrow{\text{PQ}}$;

(ii) $\overrightarrow{\text{PR}}$.

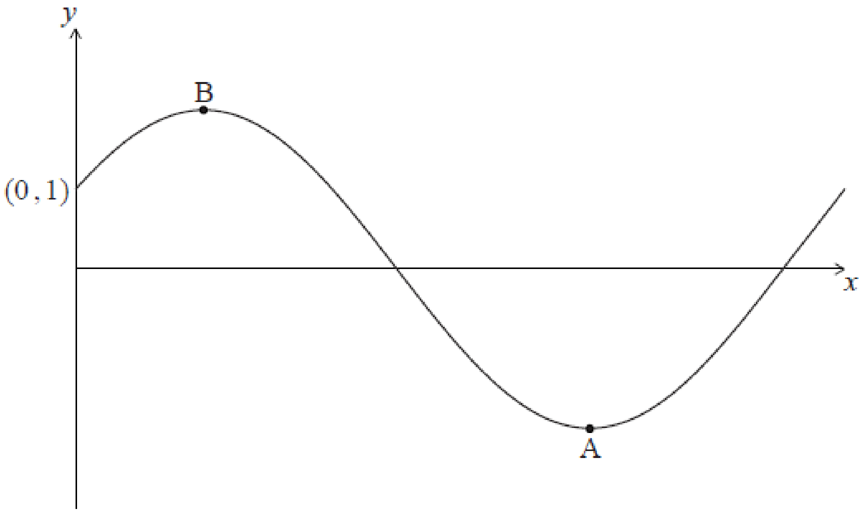
b. Show that $\cos \text{R}\hat{\text{P}}\text{Q} = \frac{1}{2}$. [7]

c. (i) Find $\sin\text{R}\hat{\text{P}}\text{Q}$. [6]

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.

Solve the equation $2\cos x = \sin 2x$, for $0 \le x \le 3\pi$.

Let $f(x) = \cos x + \sqrt{3}\sin x$, $0 \le x \le 2\pi$. The following diagram shows the graph of f .



The y-intercept is at (0, 1) , there is a minimum point at A (p, q) and a maximum point at B.

a. Find $f'(x)$. [2]

b(i) and (ii). Hence [10]

(i) show that $q = -2$;

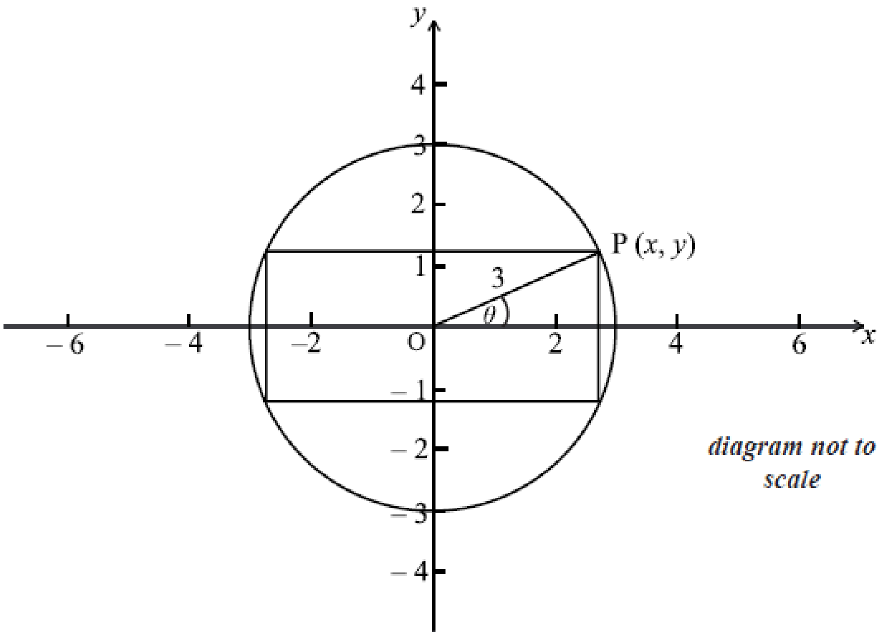
(ii) verify that A is a minimum point.

c. Find the maximum value of $f(x)$. [3]

d. The function $f(x)$ can be written in the form $r\cos(x - a)$. [2]

Write down the value of r and of a .

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(x , y) is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

a. Write down an expression in terms of θ for [2]

(i) x ;

(ii) y .

b. Let the area of the rectangle be A. [3]

Show that $A = 18 \sin 2\theta$.

c. (i) Find $\frac{\mathrm{d}A}{\mathrm{d}\theta}$. [8]

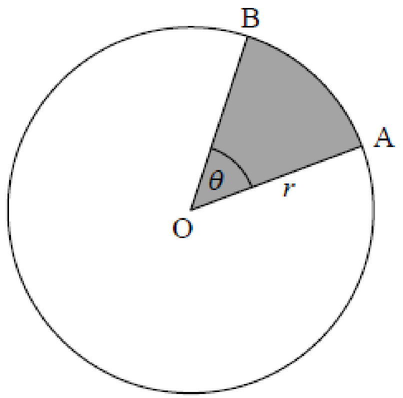
(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum.

Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$.

The following diagram shows a circle with centre O and radius *r* cm.

diagram not to scale



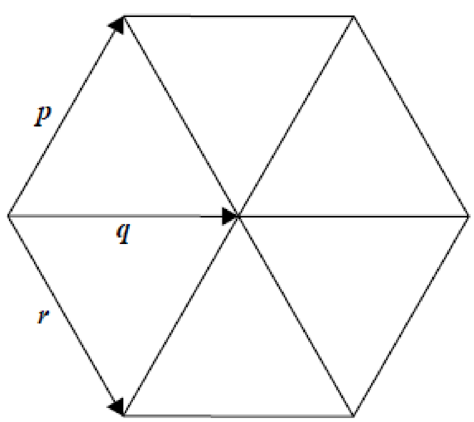
The points A and B lie on the circumference of the circle, and $A\hat{O}B = \theta$. The area of the shaded sector AOB is 12 cm^2 and the length of arc AB is 6 cm.

Find the value of *r*.

Six equilateral triangles, each with side length 3 cm, are arranged to form a hexagon.

This is shown in the following diagram.

diagram not to scale



The vectors ***p*** , ***q*** and ***r*** are shown on the diagram.

Find $\boldsymbol{p} \cdot (\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$.